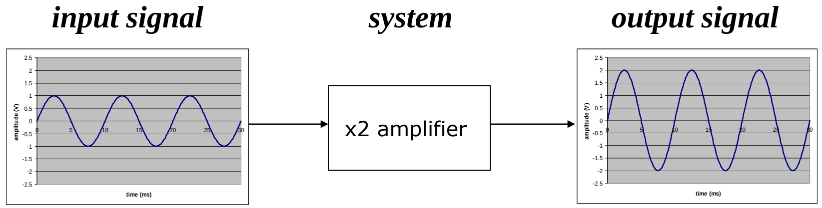
Signals & Systems for Speech & Hearing

Week 3

The **BIG** idea
&
Frequency responses of LTI Systems

A simple system



Our goal

To characterise the behaviour of a system that allows us to predict the output of the system to any input signal

Our motto

We don't care how a system changes a signal, we only care for what the system does to the signal.

→

We don't study the system itself but we compare the input to the output.

LTI systems are...

... linear

- Homogeneity
  - The amplitude of output signals grows proportionally with the amplitude of input signals, with no change in the *shape* of the output
- Additivity
  - The output to the sum of two input signals is the sum of the outputs to the two inputs separately
  - Signals don't interact

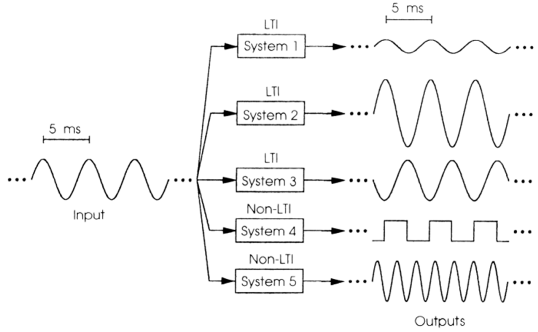
... time-invariant

- What a system does to an input signal today, is the same as what it will do tomorrow
- The system does not change its behaviour over time

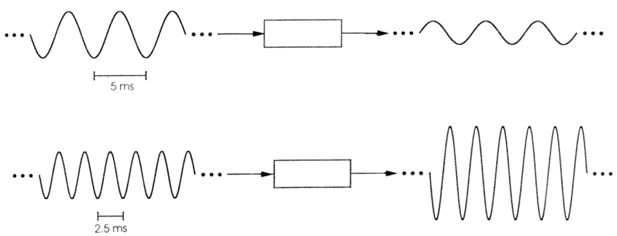
An LTI system can be completely characterised by its response to sinusoids

NEVER forget:

Sinusoidal input signals to an LTI system always lead to sinusoidal outputs of the **same frequency**

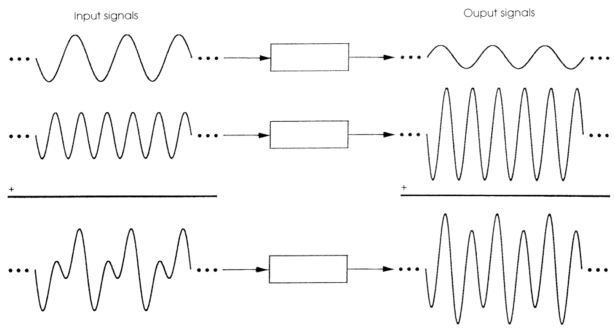


Knowing the response of a system to a sinusoid of a particular frequency, amplitude and phase allows the prediction of the output of the system to a sinusoid of the same frequency, but any amplitude and any phase



Why?

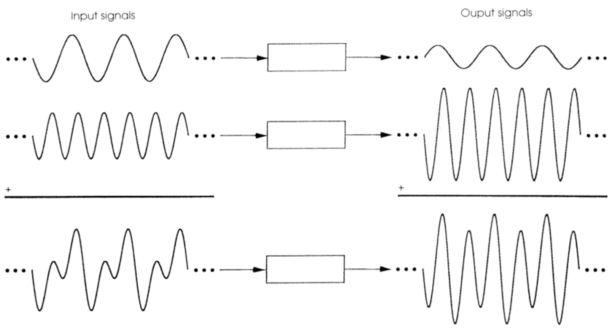
Knowing the response of a system to any frequency sinusoid allows the prediction of the output of the system to any signal that can be made from adding up sinusoids of *any* frequency, amplitude and phase



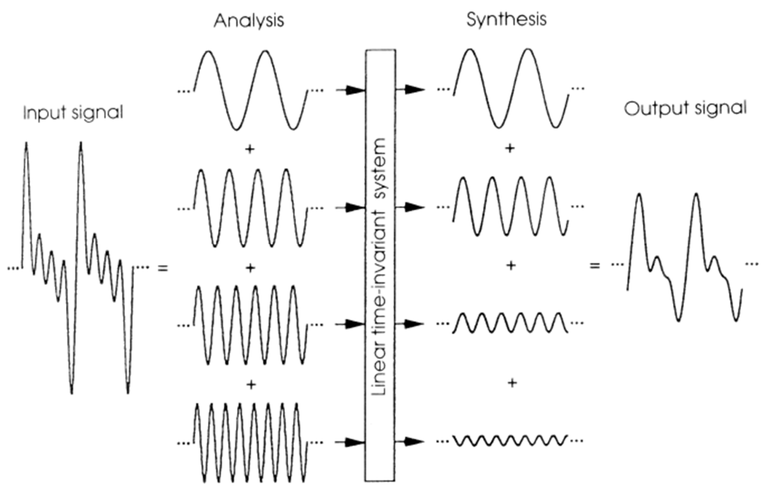
Why?

Remember:

Any complex wave can be made by adding up sinusoids of varying frequency, amplitude and phase



The BIG idea: Illustrated



Physical systems react differently to different frequencies

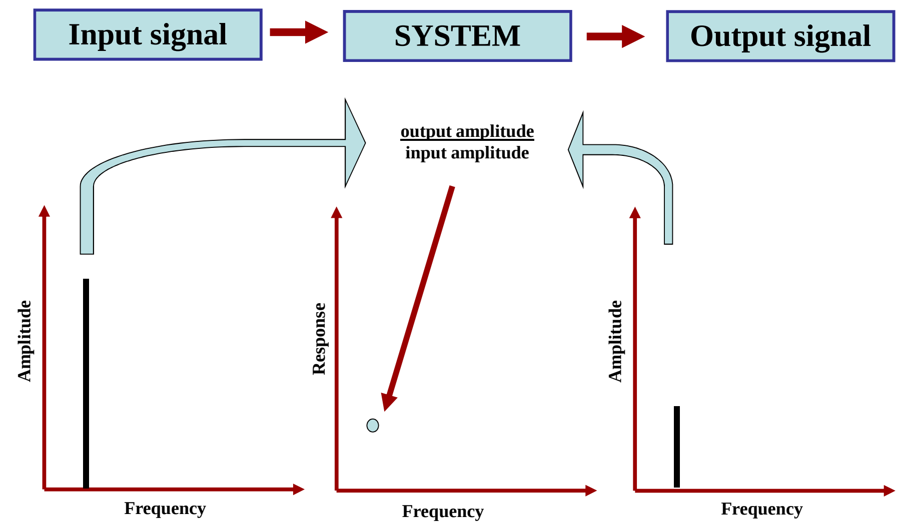
- A swing or pendulum
- Acoustic resonators
- Mass on a spring
- Bridges

thebridge_open_high.wmv

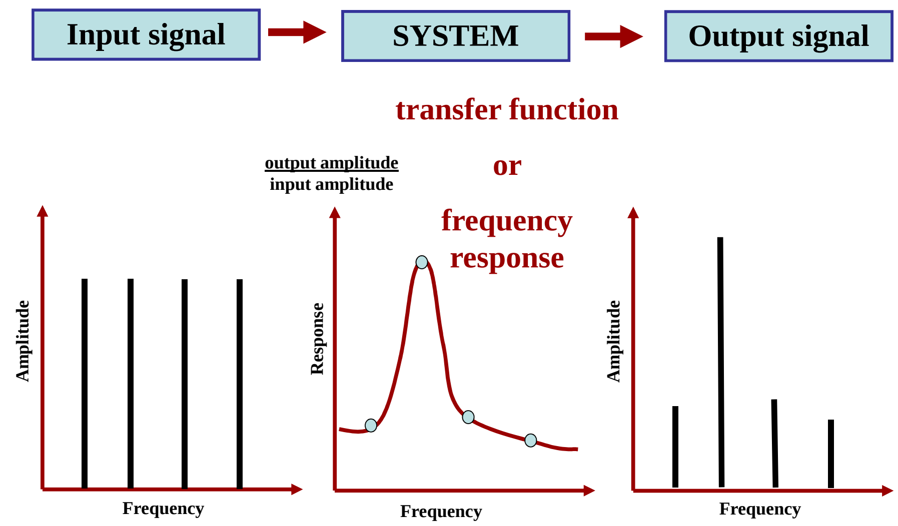
Frequency response

- Also known as a *transfer function*
- Sinusoids vary on 3 parameters
  - frequency, amplitude & phase
- For a system, we need to specify its effect on two of those
  - amplitude response
  - phase response
- Amplitude response typically more important …
  - but phase matters in certain situations

Characterisation of LTI-Systems



Characterisation of LTI-Systems

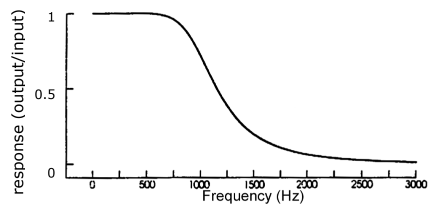


Using sinusoids to measure an amplitude response in an LTI system

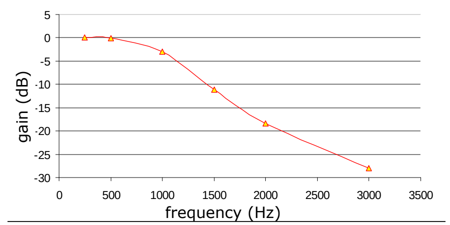
- Typically, choose a constant level for input (not necessary)
- For each frequency – feed the input sinusoid to the system and measure level at output
- Calculate the *response*
  - *R = output amplitude/input amplitude*
  - Also known as *gain*
- Need enough frequencies to map out amplitude response over frequency range of interest
- Then, for any particular frequency
  - *output amplitude = response x input amplitude*
  - Why?

At least 3 ways to specify a frequency response

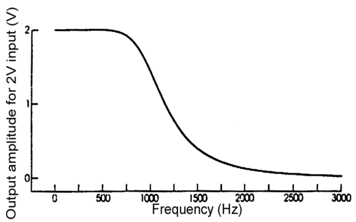
frequency (Hz)	input (V)	output (V)	amplitude ratio (re 2V input)	gain in dB
250	2	2	1	0.0
500	2	1.98	0.99	-0.1
1000	2	1.42	0.71	-3.0
1500	2	0.56	0.28	-11.1
2000	2	0.24	0.12	-18.4
3000	2	0.08	0.04	-28.0

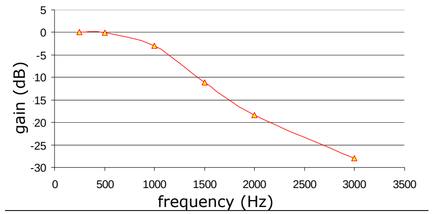


But easiest to see the overall effect on a graph, e.g. a low-pass response



Amplitude Response: Key points



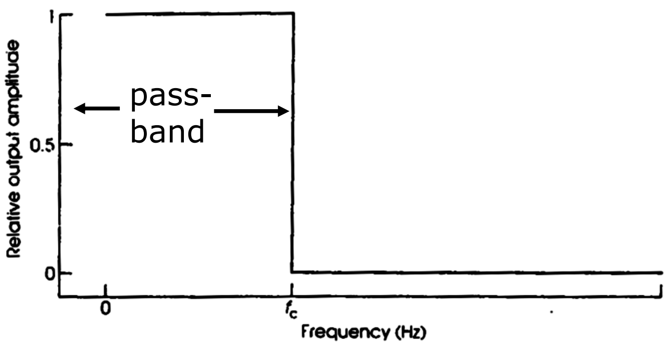


- Change made by system to amplitude of a sinewave – specified over a range of frequencies.
- Response = output amplitude/input amplitude
- Usually scaled in dB as:
  20 x log(output amplitude/input amplitude)
  = response (dB re input amplitude)

Filters

- Common name for systems that change amplitude and/or phase of waves
  - or just any LTI system
- Simple filters – low-pass and high-pass

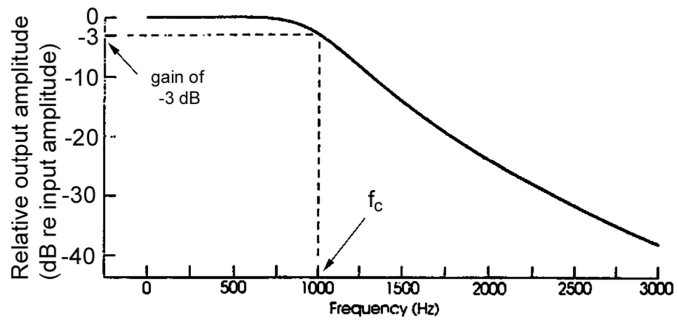
An ideal low-pass filter



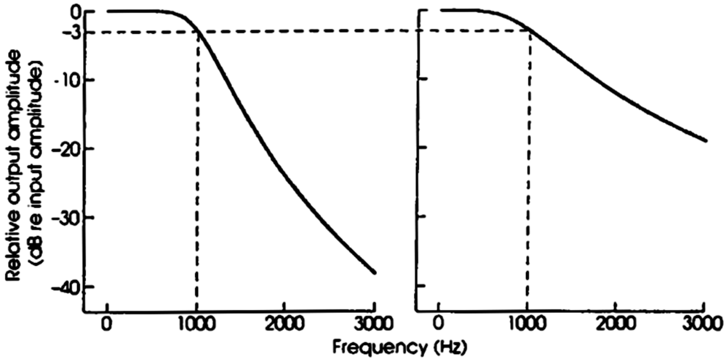
Sudden change from gain of 1 to a very small value (virtually no output at all) at cut-off frequency f_c

A realistic low-pass filter

- Defined as frequency where gain is -3dB.
- –3 dB is equivalent to half-*power* not half-*amplitude*
  10 log(0.5) = -3.0



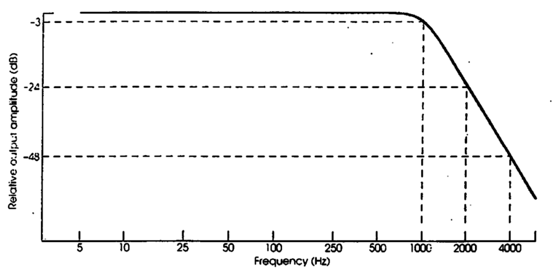
Filters can vary in shape



Slope of filter

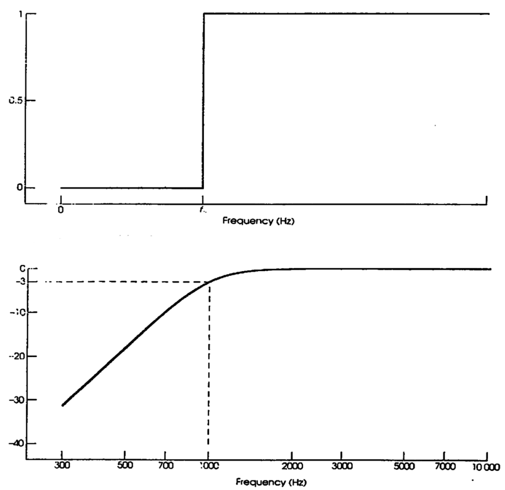
- Often constant in dB for a given frequency ratio
  - e.g., –6 dB per octave (doubling of frequency)
- suggests the use of a log frequency scale as well as a log amplitude ratio scale
  - dB in log base 10 (10, 100, 1000, etc.)
  - octave scale is log base 2, as implied in the frequency scale of an audiogram (125, 250, 500, 1000, 2000, etc).

Filter slope – in dB/octave



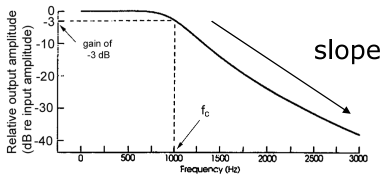
- Degrees of steepness of slope less than18 dB/octave can be called "shallow"
- 48 dB/octave or more can be called "steep"

High-pass filters



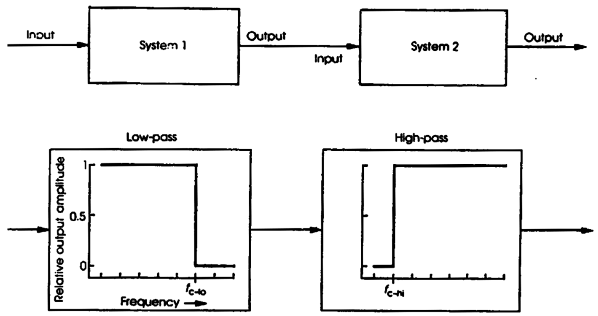
Simple filters: Key points

- High-pass or low-pass characteristics
- Defined by
  - cut-off frequency and slope of response
- Have a listen!
  - Almost all natural sounds a mixture of frequencies



Systems in cascade

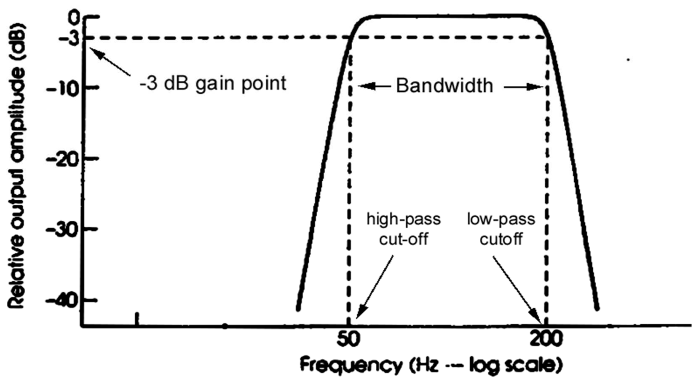
- Each stage acts independently, on the output of the previous stage



Systems in cascade

- On a linear response scale:
  - Overall amplitude response is ***product*** of component responses (*e.g.*, multiply the amplitude responses)
- On a dB (logarithmic) response scale
  - Overall amplitude response is the ***sum*** of the component responses (*i.e.*, sum the amplitude responses) …
  - Because taking logarithms turns multiplication into addition

Describing the width of a band-pass filter



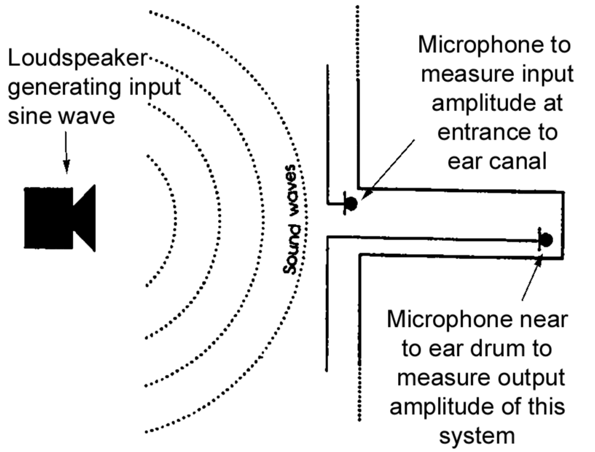
Here bandwidth (BW) is 150 Hz

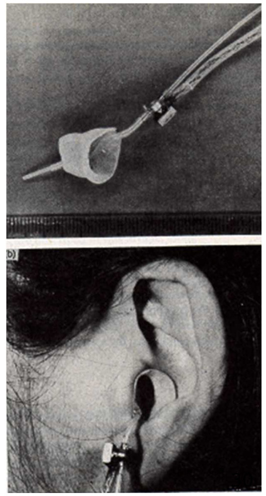
Natural filters

- Pendulum
- A relevant acoustic example:
  - a cylinder or tube closed at one end and open at the other
  - *e.g.* the ear canal

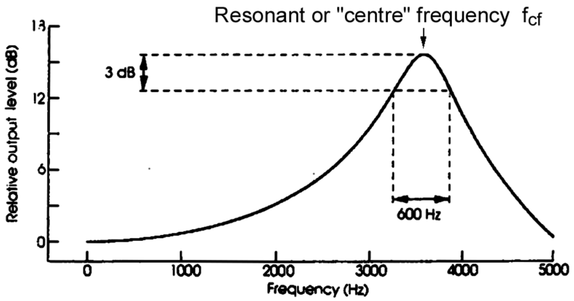
The ear canal

An acoustic tube closed at one end and open at the other (≈23 mm long)





Resonance



- Tubes like the ear canal form a special type of simple filter …
  - a resonator – similar to a band-pass filter
- Response not defined by independent high-pass and low-pass cutoff frequencies, but from a single centre frequency (the resonant frequency)
  - Resonant frequency is determined by physical characteristics of the system, often to do with size.
  - Bandwidth measured at 3 dB down points …
  - determined by the damping in the system
  - more damping=broader bandwidth

What is damping?

- The loss of energy in a vibrating system, typically due to frictional forces
- A child on a swing: feet up or brushing the floor
- A pendulum with or without a cone over the bob.
- An acoustic resonator (like the ear canal) with or without gauze over its opening

Today's lab:
Measuring the frequency response of an acoustic tube

Remember …

- All we need to know is the response of a system to sinusoids.
- An LTI system does not change the shape or frequency of a sinusoid.
- So it can only change phase or amplitude.
- Amplitude changes are usually more important, so we focus on those.
- We need to measure a so-called *amplitude response.*
  - How a system changes the amplitude of sinusoids
  - frequency response/transfer function/amplitude response

Using sinusoids to measure an amplitude response in an LTI system

- Typically, choose a constant level for input (not necessary)
- For each frequency – feed the input sinusoid to the system and measure level at output
- Calculate the *response* = *output/input*
  - Also known as *gain*
- Need enough frequencies to map out amplitude response over frequency range of interest

Scaling the response

- Generally use a logarithmic scale for response (dB) rather than linear
- Amplitude ratio expressed in dB

  =20 x log(output amp/input amp)
- Note similarity to dB SPL
  - 20 log (? Pa/20 x 10^{-6} Pa)
- Expresses output level in dB re input level

Frequency (Hz)	input (V)	output (V)	amplitude ratio (re 2V input)	gain in dB
250	2	2	1	0.0
500	2	1.98	0.99	-0.1
1000	2	1.42	0.71	-3.0
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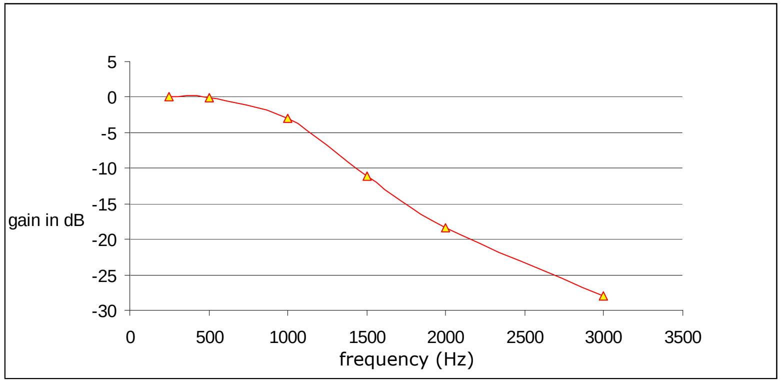
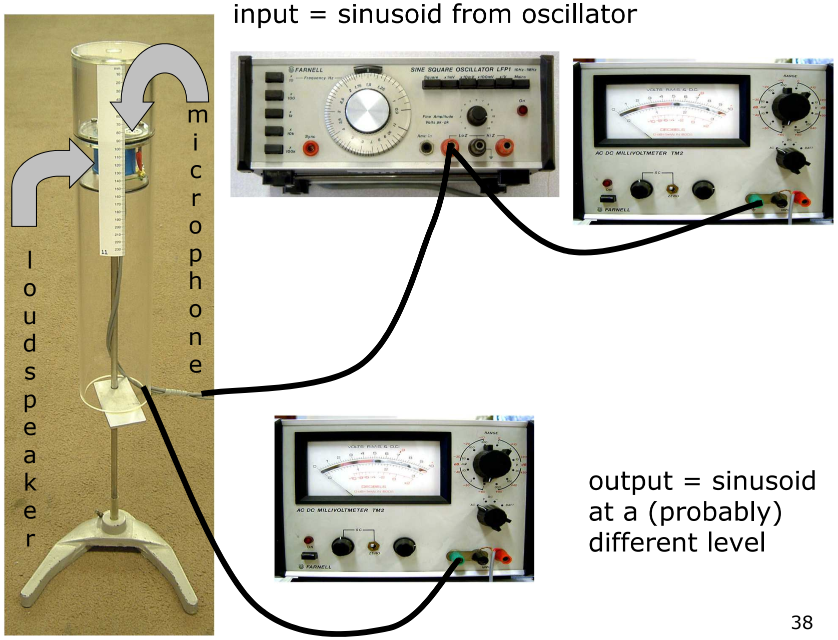


input = sinusoid from oscillator



output = sinusoid at a (probably) different level